LAB 6 – INS/GNSS integration applied to a circular trajectory (3 weeks)

Objective:

Determine the two-dimensional navigation states (position, velocity, and heading) of a 'locomotive' moving along a circular trajectory by optimally combining data from two onboard navigation instruments: an inertial measurement unit (IMU) and a GNSS receiver.

Situation:

The IMU comprises two accelerometers (tangential and radial) and a single gyroscope with vertical input axis (as in Labs 2 & 3). The IMU sensor provides readings at 100 Hz. The GNSS receiver output position fixes (as in Lab 5) at a 0.5 Hz and its antenna is assumed to coincide with the origin of the body frame of the IMU. The computational results are desired in a two-dimensional mapping (*m*-) frame with the first and second axis pointing towards the north and towards the east, respectively. The *m*-frame is supposed to be inertial frame.

Tasks:

1. Generate a reference trajectory of the virtual vehicle along the circular path (using error-free polar coordinates). Use the following motion parameters: radius r = 500 m, angular rate $\dot{\phi}$ = π/100 rad/s., initial values: $\boldsymbol{p}_0 = [r, 0]$, $\boldsymbol{v}_0 = [0, \dot{\phi} \cdot r]$.

2. As in Lab 2, simulate the nominal two-dimensional inertial measurements along the track. Similarly to Lab 3, corrupt the nominal readings of the gyroscopes by errors that are simulated as a combination of a random-constant bias, a 1st order Gauss-Markov (GM1) process and white noise as specified in Table 1. Further, corrupt the output data of each accelerometer by a time-correlated bias that is also simulated as a GM1 process as specified in Table 1.

Error type	Notation	Stochastic par.	Par. value	Initial value	Note
Gyro random bias	b_c	-	-	-400°/h	constant value
Gyro GM1	b_g	σ_G	0.01°/s/√Hz	0	PSD level
		$1/\beta_G$	30 s	-	correlation time
Gyro white noise	b_w	σ_{Gw}	0.1°/√h		white noise
Accelerometer 1&2 white noise	b_{A1w} & b_{A2w}	σ_{Aw}	50 µg/√Hz	0	white noise
Accelerometer 1 GM1	b_{A1}	σ_{A1}	200 µg/√Hz	–100 µ-g	PSD level
		$1/\beta_{A1}$	60 s	-	correlation time
Accelerometer 2 GM1	b_{A2}	σ_{A2}	200 µg/√Hz	+200 µ-g	PSD level
		$1/\beta_{A2}$	60 s	-	correlation time

Table 1: IMU sensor error characteristics for **simulation**

3. As in Lab 5, simulate GPS-position fixes along the reference path by adding Gaussian white noise ($\sigma_n^{GPS} = \sigma_e^{GPS}$ = **1.0** m) to the reference trajectory (separately for each coordinate) at **2 (two) second** time steps.

4. Calculate the Kalman-filtered trajectory based on the GPS/INS optimally integrated observations. Simulate the error in INS initialization as: +3° misalignment error, initial velocity error -2 m/s and -1 m/s in north and east components while taking the initial position as the first GPS position, i.e.

$$X_{INS} = [\,\alpha(0), v^n(0).\,v^e(0), p^n(0), p^e(0)] = [90^o + 3^o, 0 - 2, \dot{\phi}.\,r - 1, p_n^{gps}(0), p_e^{gps}(0)]$$

5. Setup a 'differential' Kalman Filter with error state, ***dx***; thereby the filter states represent the errors in the navigation states as well as systematic influences in the inertial data. For the initial filter variance assume the uncertainties as specified in Table 2.

State Variable	Initial Value	Initial Uncertainty (1 σ)
$d\alpha$	0	2°
dv^n	0	5 m/s
dv^e	0	5 m/s
dp^n	0	10 m
dp^e	0	10 m
b_c	0	0.05 °/s
b_g	0	0.01°/s
b_{A1}	0	300 μg
b_{A2}	0	300 μg

Table 2: Parameters for initial filter state ***dx***(t=0) and its covariance *P*(t=0).

- IMU error states: use true stochastic values for GM1 processes (b_G, b_{A1}, b_{A2}) according to Table 1.

6. In the report discuss the following points:

- The accuracy of the INS (before and after) the 1st and 10th GPS-measurement updates.
- Comment on the empirical and predicted Kalman filtered positioning accuracy.
- The influence of setting the correlation times in the GM1 models used by the Kalman Filter incorrect (increase/decrease by a factor of 100).

Deliverables:

1. Pseudo-code: Max. 1 page. More information on Peudocode can be found over Wikipedia (https://en.wikipedia.org/wiki/Pseudocode).
2. Plot the errors in the navigation states and 3-sigma bounds (from the covariance matrix). Comment on the obtained results
3. Plot the IMU error states and 3-sigma bounds (from the covariance matrix). Comment on the obtained results
4. MATLAB Code
5. Answers to questions in Task 6

Lab weight: 5%
Deadline: Pseudocode before 23/05/2025 (12:00h) and full lab by 08/06/2025 (23:59h)